Quiz #2 **Math 181** *6/30/21* **Name:** KEY

Show all your work clearly. No Work, No Credit.

1. Integrate the following:

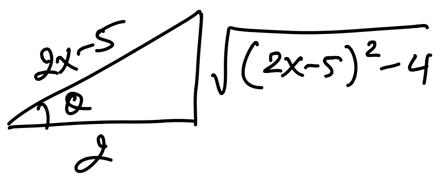
a) $\displaystyle\int \frac{2x-1}{\sqrt{4x^2-20x+21}}\,dx$

$$4x^2-20x+21 = \underbrace{(2x)^2-2(2x)\cdot 5+25}-4 = (2x-5)^2-4$$

$$= \int \frac{2x-5+4}{\sqrt{(2x-5)^2-4}}\,dx \qquad \left\{\begin{array}{l} \text{let } 2x-5 = 2\sec\theta \\ 2dx = 2\sec\theta\tan\theta\,d\theta \\ dx = \sec\theta\tan\theta\,d\theta \end{array}\right.$$

$$= \int \frac{2\sec\theta+4}{\sqrt{4\sec^2\theta-4}}\cdot\sec\theta\tan\theta\,d\theta = 2\int\frac{\sec\theta+2}{2\cdot\tan\theta}\cdot\sec\theta\tan\theta\,d\theta$$

$$= \int(\sec\theta+2)\sec\theta\,d\theta = \int(\sec^2\theta+2\sec\theta)\,d\theta = \tan\theta+2\ln|\sec\theta+\tan\theta|+C$$



$$= \frac{\sqrt{(2x-5)^2-4}}{2}+2\ln\left|\frac{2x-5}{2}+\frac{\sqrt{(2x-5)^2-4}}{2}\right|+C$$

b) $\displaystyle\int \frac{x^2+5x+32}{(x+5)(x+1)^2}\,dx = \int\left(\frac{A}{x+5}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\right)dx.$

$$A\Big|_{x=-5} = \frac{25-25+32}{16} = 2.\ ;\quad C\Big|_{x=-1} = \frac{1-5+32}{4} = 7$$

$$\text{plug } x=0 \Rightarrow \frac{32}{5} = \frac{A}{5}+B+C = \frac{2}{5}+B+7$$

$$\Rightarrow B = \frac{32}{5}-\frac{2}{5}-7 = -1$$

$$= \int\left[\frac{2}{x+5}-\frac{1}{x+1}+\frac{7}{(x+1)^2}\right]dx$$

$$= 2\ln|x+5|-\ln|x+1|-\frac{7}{x+1}+C.$$

2. Sketch and then set up integral(s) for volumes of the region bounded by the curves: $x = 2y^2 - 5y - 3$ *and* $x = 4y + 2$ which is rotated about the following lines:

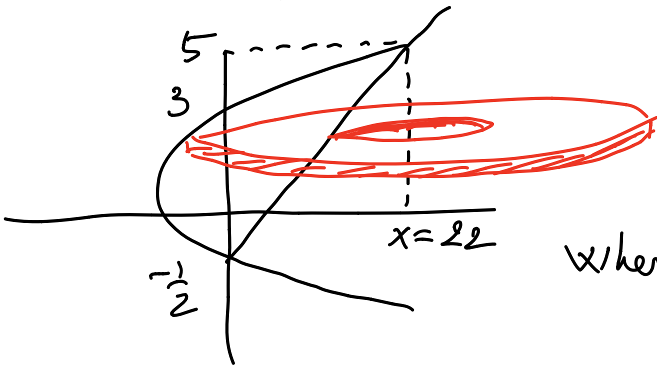
a) $x = 22$

$x = (2y+1)(y-3)$

pts of intersection: $2y^2 - 5y - 3 = 4y + 2$

$2y^2 - 9y - 5 = (2y+1)(y-5) = 0$

$y = -\frac{1}{2}, 5$



$\Rightarrow V = \pi \left[R_o^2 - r_i^2 \right] dy$.

where $R_o = = 22 - (2y^2 - 5y - 3) = 25 - 2y^2 + 5y$

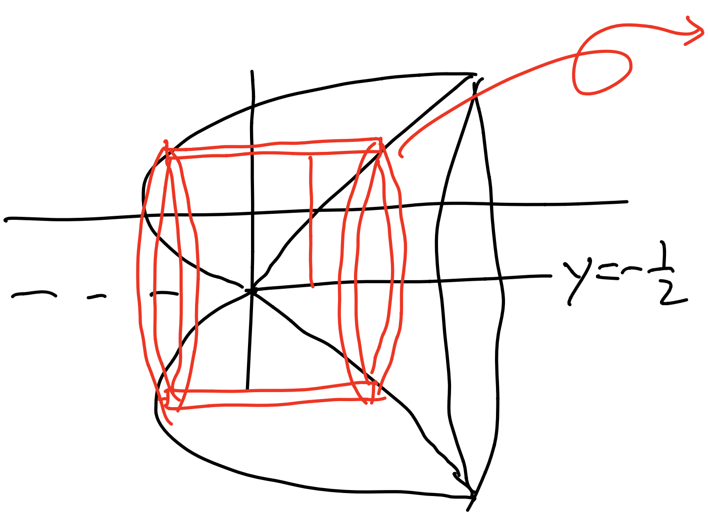
$x = 2y^2 - 5y - 3$ $\quad x = 22$

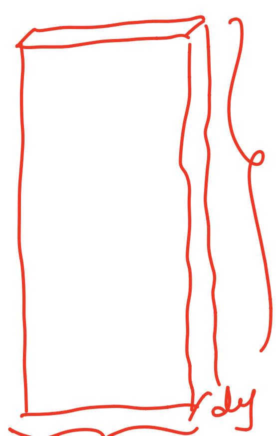
$r_i = = 22 - (4y+2) = 20 - 4y$

$x = 4y + 2$ $\quad x = 22$

$$\Rightarrow V = \pi \int_{-1/2}^{5} \left[(25 + 5y - 2y^2)^2 - (20 - 4y)^2 \right] dy.$$

b) $y = -\frac{1}{2}$



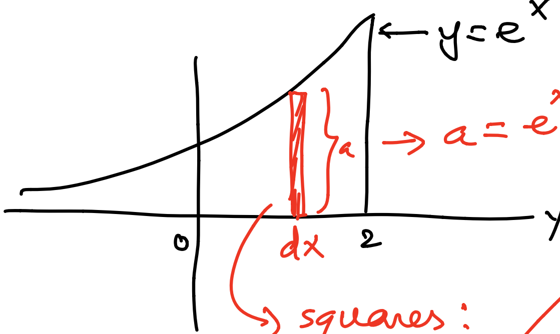


$w = (4y+2) - (2y^2 - 5y - 3)$

$= 9y + 5 - 2y^2$

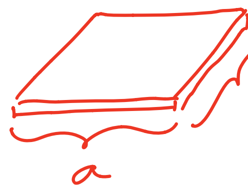
$$\Rightarrow V = 2\pi \int_{-1/2}^{5} \left(y + \frac{1}{2}\right)\left(9y + 5 - 2y^2\right) dy$$

3. The base of a solid is bounded by $y = e^x$ *and* $y = 0$ *for* $0 \le x \le 2$. All parallel – cross – sections are squares perpendicular to the base and the x – axis. Find its volume.



$\rightarrow a = e^x - 0 = e^x$

squares:



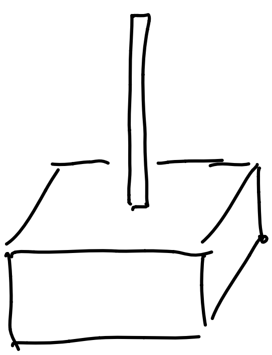
$\Rightarrow V = a^2\,dx$

$$V = \int_0^2 (e^x)^2\,dx$$

$$\Rightarrow V = \int_0^2 e^{2x}\,dx = \frac{1}{2}e^{2x}\Big|_0^2 = \frac{1}{2}\left[e^4 - e^0\right]$$

Ans: $= \frac{1}{2}\left[e^4 - 1\right] = \underline{26.79}$.

4. A 500 – ft – cable that weights 600lbs is attached to a 1500 – lbs – elevator. Calculate the total work required to pull both the cable and the elevator up 500 ft. (10pts)



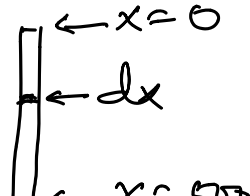
For the elevator:

$$W_e = F\cdot d = (1500\,\text{lbs})(500\,\text{ft})$$
$$= 750{,}000\ \text{ft-lbs}.$$

For the cable:

Length	Force
500 ft	600 lb.
1 ft	$\rightarrow \frac{600}{500} = \frac{6}{5}$ lb.
⋮	
dx	$\rightarrow \frac{6}{5}dx$.

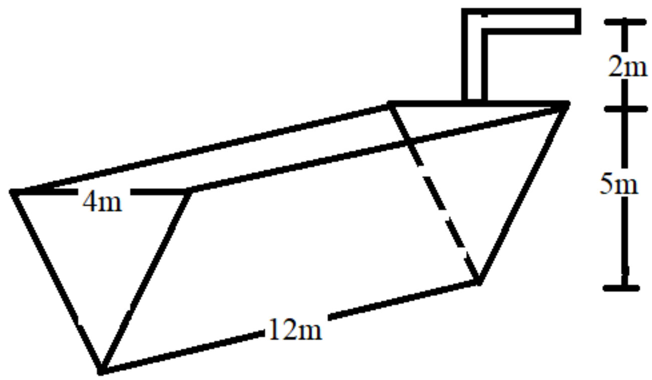
distance $d = x$



$$\Rightarrow W_c = \int_0^{500} \frac{6}{5}x\,dx = \frac{3}{5}x^2\Big|_0^{500} = \frac{3}{5}(500)^2 = 150{,}000\ \text{ft-lbs}.$$

$\Rightarrow$ Total work: $W = W_e + W_c = 750{,}000 + 150{,}000 = 900{,}000$ ft-lbs.

5. The following tank is full of water. Determine the work required to pump the water out of the outlet.



Check the answer from the other Math 181 class.