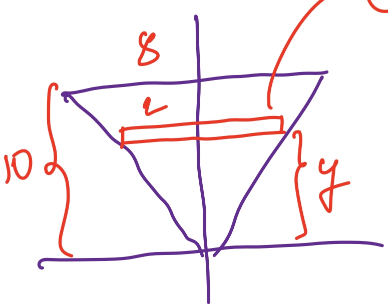
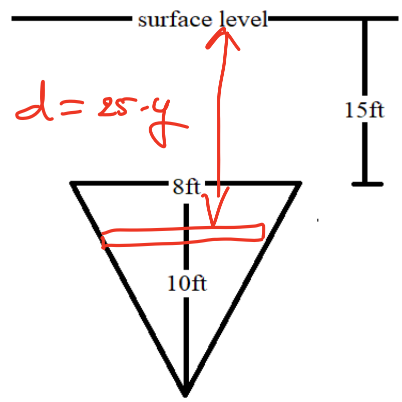
Quiz #3 ***Math 181*** ***Name:***

7/14/21

Show all your work clearly. No Work, No Credit.

1. The isosceles triangular plate shown as follows is submerge vertically 15 ft below the surface of water. Determine the fluid force against one face of the plate. (10 pts)





dx

$L = \frac{4}{5}y$

$\frac{L}{8} = \frac{y}{10} \Rightarrow L = \frac{4}{5}y$

$A = \frac{4}{5}y\,dy$

$d = 25-y$.

$V = \frac{4}{5}y(25-y)\,dy$

$F = (62.5)\left(\frac{4}{5}\right)\int_0^{10} y(25-y)\,dy$.

$= 50\int_0^{10}(25y-y^2)\,dy$

$= 50\left[\frac{25}{2}(10)^2 - \frac{1}{3}(10)^3\right] = 45,833 \text{ lbs.}$

2. Solve the following differential equations: (10 pts)

a) $\dfrac{dy}{dx} = \dfrac{e^{2-3x-4y}}{e^{4x+y+1}}$

$$\frac{dy}{dx} = e^{2-3x-4y-4x-y-1} = e^{1-7x-5y} = e^{1-7x} \cdot e^{-5y}$$

$$\frac{dy}{e^{-5y}} = e^{1-7x}\,dx \Rightarrow \int e^{5y}\,dy = \int e^{1-7x}\,dx$$

$$\Rightarrow \frac{1}{5}e^{5y} = -\frac{1}{7}e^{1-7x} + C \Rightarrow e^{5y} = -\frac{5}{7}e^{1-7x} + C$$

$$\Rightarrow \underline{Sol}: y = \frac{1}{5}\ln\left(C - \frac{5}{7}e^{1-7x}\right).$$

b) $\dfrac{dy}{dx} = 6x^2y^3 - 2xy^3 + y^3;\ y(0) = 1$

$$\frac{dy}{dx} = (6x^2 - 2x + 1)y^3 \Rightarrow \frac{dy}{y^3} = (6x^2 - 2x + 1)dx$$

$$\Rightarrow \int y^{-3}\,dy = \int (6x^2 - 2x + 1)\,dx$$

$$-\frac{1}{2}y^{-2} = 2x^3 - x^2 + x + C.$$

$$y^{-2} = -4x^3 + 2x^2 - 2x + C = \frac{1}{y^2}$$

$$\Rightarrow y = \pm\sqrt{\frac{1}{C - 4x^3 + 2x^2 - 2x}}.$$

3. Calculate the arc-length of the curve: $\begin{cases} x = e^{2t}\sin(3t) \\ y = e^{2t}\cos(3t) \end{cases}$; *for* $0 \le t \le \frac{\pi}{6}$ (10 pts) $\Rightarrow$ $L = \int ds.$

where $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$\frac{dx}{dt} = e^{2t}\left[2\sin(3t) + 3\cos(3t)\right] \Rightarrow \left(\frac{dx}{dt}\right)^2 = e^{4t}\left(4\sin^2(3t) + 12\sin(3t)\cos(3t) + 9\cos^2(3t)\right)$

$\frac{dy}{dt} = e^{2t}\left[2\cos(3t) - 3\sin(3t)\right] \Rightarrow \left(\frac{dy}{dt}\right)^2 = e^{4t}\left(4\cos^2(3t) - 12\cos(3t)\sin(3t) + 9\sin^2(3t)\right)$

$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{4t}(4+9) = 13e^{4t}.$

$\Rightarrow L = \int_0^{\pi/6} \sqrt{13e^{4t}}\,dt = \sqrt{13}\int_0^{\pi/6} e^{2t}\,dt$

$= \frac{\sqrt{13}}{2} e^{2t}\Big|_0^{\pi/6} = \frac{\sqrt{13}}{2}\left[e^{\frac{\pi}{3}} - 1\right] = \#.$

4. Find surface area of the curve: $y = \frac{1}{3}\left(x^2+2\right)^{3/2}$ *for* $0 \le x \le \sqrt{2}$ rotated about the y– axis. (10 pts)

$S = 2\pi\int r\,ds$ where $\begin{cases} ds = \sqrt{1+(y')^2}\,dx \\ r = x \end{cases}$ match.

$y' = \frac{1}{2}(x^2+2)^{1/2}\cdot 2x = x\sqrt{x^2+2} \Rightarrow (y')^2 = \left(x\sqrt{x^2+2}\right)^2 = x^2(x^2+2)$

$\Rightarrow 1 + (y')^2 = x^4 + 2x^2 + 1 = (x^2+1)^2.$

$\Rightarrow S = 2\pi\int_0^{\sqrt{2}} x\sqrt{(x^2+1)^2}\,dx = 2\pi\int_0^{\sqrt{2}} x(x^2+1)\,dx$

$= 2\pi\int_0^{\sqrt{2}} (x^3+x)\,dx = 2\pi\left[\frac{1}{4}x^4 + \frac{1}{2}x^2\right]_0^{\sqrt{2}}$

$= 2\pi\left(\frac{1}{4}(\sqrt{2})^4 + \frac{1}{2}(\sqrt{2})^2\right] = 2\pi(1+1)$

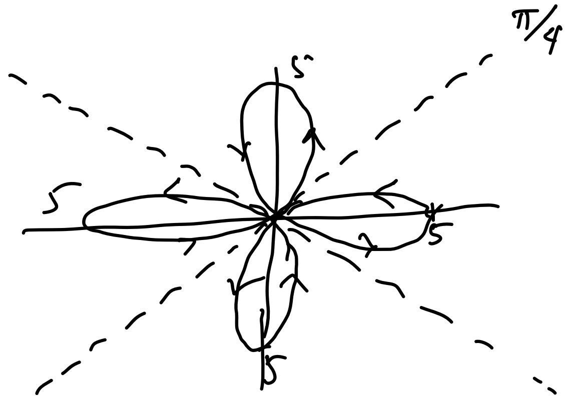
$= \boxed{4\pi}$

5. Sketch the graph of the following functions in polar coordinates: (10 pts)

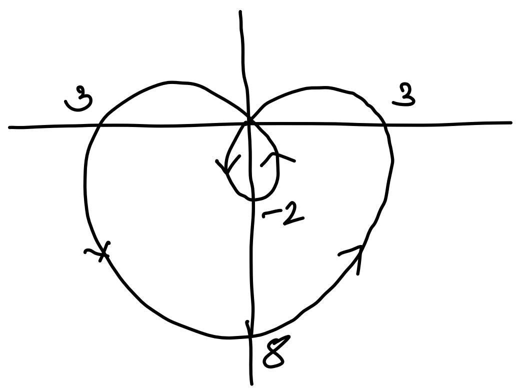
a) $r = 5\cos(2\theta)$

$\theta = 0 \Rightarrow r = 5$

$\theta = \frac{\pi}{4} \Rightarrow r = 0$



b) $r = 3 - 5\sin(\theta)$



c) $r = 4 + 4\cos(\theta)$