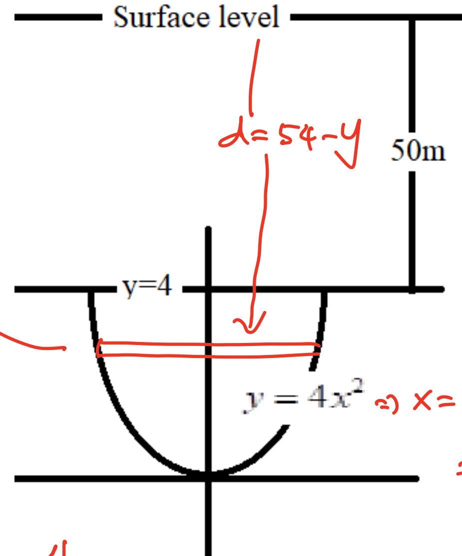
Quiz #3 **Math 181** **Name:** KEY

7/14/21

Show all your work clearly. No Work, No Credit.

1. A parabolic plate bounded by $y = 4x^2$ *and* $y = 4$ shown as follow is submerged vertically 50m below the surface of water. Determine the fluid force against one face of the plate. (10 pts)



$y = 4x^2 \Rightarrow x = \pm\sqrt{\tfrac{1}{4}y} = \pm\tfrac{1}{2}\sqrt{y}$

$\underbrace{\boxed{}}_{L\,=\,\sqrt{y}}\ dy$

$L = x_r - x_L = \frac{1}{2}\sqrt{y} + \frac{1}{2}\sqrt{y} = \sqrt{y}$

$\Rightarrow A = \sqrt{y}\,dy$

$d = 54 - y$

$V = \sqrt{y}\,(54-y)\,dy$

$$F = 9800\int_0^4 \sqrt{y}\,(54-y)\,dy = 9800\int_0^4 \left(54y^{1/2} - y^{3/2}\right)dy$$

$$= 9800\left[54\cdot\frac{2}{3}\,y^{3/2} - \frac{2}{5}\,y^{5/2}\right]_0^4$$

$$= 9800\left[36\,(4)^{3/2} - \frac{2}{5}\,(4)^{5/2}\right]$$

$$= 9800\left[288 - \frac{64}{5}\right] = 2{,}696{,}960\ N.$$

2. Solve the following differential equations: (10 pts)

a) $\dfrac{dy}{dx}=\dfrac{3y-5}{2x+3}$ $\Rightarrow \displaystyle\int \frac{dy}{3y-5} = \int \frac{dx}{2x+3}$

$$\Rightarrow \frac{1}{3}\ln|3y-5| = \frac{1}{2}\ln|2x+3| + C.$$

$$\ln|3y-5| = \frac{3}{2}\ln|2x+3| + C$$

$$3y-5 = e^{\ln(2x+3)^{3/2}} \cdot e^{C} = K(2x+3)^{3/2}$$

$$y = K(2x+3)^{3/2} + \frac{5}{3}$$

b) $\dfrac{dy}{dx}=\dfrac{e^{y-2x+3}}{e^{x-3y+1}}$; $y(0)=1$

$$\frac{dy}{dx} = e^{y-2x+3-x+3y-1} = e^{-3x+2+4y} = e^{-3x+2}\cdot e^{4y}$$

$$\Rightarrow \int \frac{dy}{e^{4y}} = \int e^{-3x+2}\,dx$$

$$\Rightarrow \int e^{-4y}\,dy = \int e^{-3x+2}\,dx$$

$$-\frac{1}{4}e^{-4y} = -\frac{1}{3}e^{-3x+2} + C.$$

$$e^{-4y} = \frac{4}{3}e^{-3x+2} + C$$

$$y = -\frac{1}{4}\ln\left(\frac{4}{3}e^{-3x+2} + C\right)$$

$$y(0) = -\frac{1}{4}\ln\left(\frac{4}{3}e^{2} + C\right) = 1$$

$$\ln\left(\frac{4}{3}e^{2} + C\right) = -4$$

$$\frac{4}{3}e^{2} + C = e^{-4}$$

$$C = \frac{1}{e^{4}} - \frac{4}{3}e^{2}$$

$\Rightarrow$ Sol:

$$y = -\frac{1}{4}\ln\left(\frac{4}{3}e^{-3x+2} + \frac{1}{e^{4}} - \frac{4}{3}e^{2}\right)$$

3. Calculate the arc-length of the curve: $\begin{cases} x = e^{3t}\cos(2t) \\ y = e^{3t}\sin(2t) \end{cases}$; *for* $0 \le t \le \frac{\pi}{4}$ (10 pts) $\Rightarrow L = \int ds$

where $ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$.

$\frac{dx}{dt} = e^{3t}\left(3\cos(2t) - 2\sin(2t)\right) \Rightarrow \left(\frac{dx}{dt}\right)^2 = e^{6t}\left(9\cos^2(2t) - 12\sin(2t)\cos(2t) + 4\sin^2(2t)\right)$

$\frac{dy}{dt} = e^{3t}\left(3\sin(2t) + 2\cos(2t)\right) \Rightarrow \left(\frac{dy}{dt}\right)^2 = e^{6t}\left(9\sin^2(2t) + 12\cos(2t)\sin(2t) + 4\cos^2(2t)\right)$.

$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = e^{6t}\left[9+4\right] = 13e^{6t}$.

$\Rightarrow L = \int_0^{\pi/4} \sqrt{13e^{6t}}\,dt = \sqrt{13}\int_0^{\pi/4} e^{3t}\,dt = \frac{\sqrt{13}}{3} e^{3t}\Big|_0^{\pi/4}$

$= \boxed{\frac{\sqrt{13}}{3}\left[e^{\frac{3\pi}{4}} - 1\right]}$

4. Find surface area of the curve: $y = \sqrt{1+e^x}$ *for* $0 \le x \le 1$ rotated about the x– axis. (10 pts)

$S = 2\pi\int r\,ds$ where $\begin{cases} ds = \sqrt{1+(y')^2}\,dx \\ r = y = \sqrt{1+e^x} \end{cases}$

$y = \sqrt{1+e^x} \Rightarrow y' = \frac{1}{2}(1+e^x)^{-1/2}\cdot e^x \Rightarrow y' = \frac{e^x}{2\sqrt{1+e^x}} \Rightarrow (y')^2 = \left(\frac{e^x}{2\sqrt{1+e^x}}\right)^2 = \frac{e^{2x}}{4(1+e^x)}$

$\Rightarrow 1+(y')^2 = 1 + \frac{e^{2x}}{4+4e^x} = \frac{4+4e^x+e^{2x}}{4+4e^x} = \frac{(2+e^x)^2}{4(1+e^x)}$

$\Rightarrow S = 2\pi\int_0^1 \sqrt{1+e^x}\cdot\sqrt{\frac{(2+e^x)^2}{4(1+e^x)}}\,dx = 2\pi\int_0^1 \frac{\cancel{\sqrt{1+e^x}}\cdot(2+e^x)}{2\cancel{\sqrt{1+e^x}}}\,dx$

$= \pi\int_0^1 (2+e^x)\,dx = \pi\left[2x+e^x\right]_0^1 = \pi\left[2+e-1\right]$

$= \pi(1+e)$

5. Sketch the graph of the following functions in polar coordinates: (10 pts)

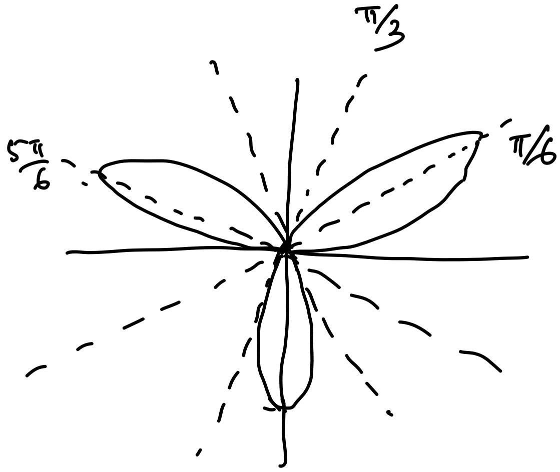
a) $r = 7\sin(3\theta)$

$\theta = 0 \Rightarrow r = 7\sin(0) = 0$

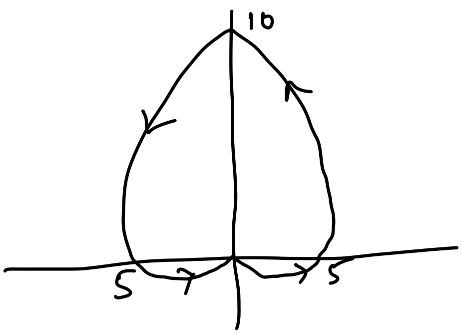
$\theta = \frac{\pi}{6} \Rightarrow r = 7\sin\left(\frac{\pi}{2}\right) = 7$

$\theta = \frac{\pi}{3} \Rightarrow r = 7\sin(\pi) = 0$

$\theta = \frac{\pi}{2} \Rightarrow r = 7\sin\left(\frac{3\pi}{2}\right) = -7$



b) $r = 5 + 5\sin(\theta)$



c) $r = 4 - 7\cos(\theta)$