Exam #2 ***Math 290*** ***Name:*** KEY

4/20/21

Show all your work clearly. No Work, No Credit.

1. ***You have 3 hours to finish this exam. Late submit exam will not be accepted.***
2. ***It's courtesy that you are the one who take this exam, please do not seek outside help of any kind, do not search internet for solutions.***
3. ***Scan your exam as one pdf file and submit it thru Canvas.***
4. ***Put your Full Name clearly on the first page.***

1. Solve the following DE: (30 pts)

a) $3y''-5y'-2y=14e^{2x}-10\sin x;\ y(0)=-3,\ y'(0)=1$

Homogeneous: $p(\lambda)=3\lambda^2-5\lambda-2=(3\lambda+1)(\lambda-2)=0 \Rightarrow \lambda=-\frac{1}{3}, 2.$

$$y_h = c_1e^{-\frac{1}{3}x}+c_2e^{2x}$$

$\{-2\}\ y_{p_1} = Axe^{2x}$

$\{-5\}\ y'_{p_1} = Ae^{2x}[2x+1]$

$\{3\}\ y''_{p_1} = Ae^{2x}[4x+2+2]$

$\Rightarrow Ae^{2x}\left[-2x-10x+12x-5+12\right] = 7Ae^{2x} = 14e^{2x}$
$\Rightarrow A=2.$

$\Rightarrow y_{p_1} = 2xe^{2x}$

$\{-2\}\ y_{p_2} = A\cos(x)+B\sin(x)$

$\{-5\}\ y'_{p_2} = -A\sin(x)+B\cos(x)$

$\{3\}\ y''_{p_2} = -A\cos x - B\sin(x)$

$\cos(x)\left[-2A-5B-3A\right]+\sin(x)\left[-2B+5A-3B\right]$

$=(-5A-5B)\cos(x)+(5A-5B)\sin(x) = -10\sin(x)$

$$\Rightarrow \begin{cases} -5A-5B=0 \\ 5A-5B=-10 \end{cases}$$
$$-10B=-10$$

$\rightarrow B=1 \Rightarrow -5A-5=0$
$A=-1$

$\Rightarrow y_{p_2} = -\cos x+\sin x$

General sol: $y=c_1e^{-\frac{1}{3}x}+c_2e^{2x}+2xe^{2x}-\cos x+\sin x$; $y'(x)=-\frac{1}{3}c_1e^{-\frac{1}{3}x}+2c_2e^{2x}+2e^{2x}[2x+1]+\sin x+\cos x$

$y(0)=c_1+c_2-1=-3$;

$y'(0)=-\frac{1}{3}c_1+2c_2+2+1=1$ $\Rightarrow$
$$\begin{cases} c_1+c_2=-2 \\ -c_1+6c_2=-6 \end{cases}$$
$7c_2=-8 \Rightarrow c_2=-\frac{8}{7}$

$c_1=-2+\frac{8}{7}=-\frac{6}{7}$

Sol^n:

$\Rightarrow y=-\frac{6}{7}e^{-\frac{1}{3}x}-\frac{8}{7}e^{2x}+2xe^{2x}-\cos x+\sin x$

b) $9y''+30y'+29y=29x^2-27x+44;\ y(0)=2,\ y'(0)=-3$

Homo: $p(\lambda)=9\lambda^2+30\lambda+29=0 \Rightarrow \underbrace{\left[(3\lambda)^2+2(3\lambda)\cdot 5+25\right]}_{(3\lambda+5)^2=-4}+4=0$

$\Rightarrow \lambda=-\frac{5}{3}\pm\frac{2}{3}i$

$\Rightarrow y_h=e^{-\frac{5}{3}x}\left[c_1\cos\left(\frac{2}{3}x\right)+c_2\sin\left(\frac{2}{3}x\right)\right].$

$$\left.\begin{aligned}\{29\}\ y_p&=ax^2+bx+c\\ \{30\}\ y_p'&=2ax+b\\ \{9\}\ y_p''&=2a\end{aligned}\right\}\ 29ax^2+(29b+60a)x+(29c+30b+18a)$$

$$\Rightarrow\left\{\begin{aligned}&29a=29\Rightarrow a=1\\ &29b+60=-27\Rightarrow b=-3\\ &29c+30(-3)+18=44\Rightarrow c=4\end{aligned}\right.$$

$y_p=x^2-3x+4.$

General soln: $y=e^{-\frac{5}{3}x}\left[c_1\cos\left(\frac{2}{3}x\right)+c_2\sin\left(\frac{2}{3}x\right)\right]+x^2-3x+4$

$y(0)=c_1+4=2\Rightarrow c_1=-2$

$y'(x)=e^{-\frac{5}{3}x}\left[-\frac{5}{3}c_1\cos\left(\frac{2}{3}x\right)-\frac{5}{3}c_2\sin\left(\frac{2}{3}x\right)-\frac{2}{3}c_1\sin\left(\frac{2}{3}x\right)+\frac{2}{3}c_2\cos\left(\frac{2}{3}x\right)\right]$
$+2x-3.$

$\Rightarrow y'(0)=-\frac{5}{3}(-2)+\frac{2}{3}c_2-3=-3\Rightarrow 10+2c_2=0\Rightarrow c_2=-5$

Sol: $y(x)=e^{-\frac{5}{3}x}\left[-2\cos\left(\frac{2}{3}x\right)-5\sin\left(\frac{2}{3}x\right)\right]+x^2-x+4$

c) $y''' + 5y'' + 4y' - 10y = 1 - 10x^2$

$P(\lambda) = \lambda^3 + 5\lambda^2 + 4\lambda - 10 = 0$ since $P(1) = 1+5+4-10 = 0 \Rightarrow$

1	1	5	4	-10
		1	6	10
	1	6	10	0

$\lambda^2 + 6\lambda + 10 = 0$

$\underbrace{\lambda^2 + 6\lambda + 9}_{} + 1 = 0$

$(\lambda+3)^2 = -1$

$\Rightarrow \lambda = -3 \pm i$

$y_h = e^{-3x}(c_1 \cos x + c_2 \sin x) + c_3 e^x$

$y_p = ax^2 + bx + c \quad \{-10\}$

$y_p' = 2ax + b \quad \{4\}$

$y_p'' = 2a \quad \{5\}$

$y_p''' = 0 \quad \{1\}$

$-10ax^2 + (-10b + 8a)x - 10c + 4b + 10a = 1 - 10x^2$

$-10a = -10 \Rightarrow a = 1, \Rightarrow -10b + 8 = 0 \Rightarrow b = \frac{8}{10} = \frac{4}{5}.$

$-10c + 4b + 10a = 1 \Rightarrow -10c + \frac{16}{5} + 10 = 1$

$10c = \frac{66}{5} - 1 = \frac{61}{5} \Rightarrow c = \frac{61}{50}$

Sol: $y = e^{-3x}\left[c_1 \cos x + c_2 \sin x\right] + c_3 e^x + x^2 + \frac{4}{5}x + \frac{61}{50}$

d) $y''+y=\sec(x)+10e^{2x} \quad for \quad -\frac{\pi}{2}<x<\frac{\pi}{2}$

Homogeneous: $p(\lambda)=\lambda^2+1=0 \Rightarrow \lambda=\pm i$

$y_h = C_1\cos x + C_2\sin x \Rightarrow \begin{cases} y_1=\cos x \ ; \ y_2=\sin x. \\ \Rightarrow W[y_1, y_2] = \det\begin{bmatrix}\cos x & \sin x \\ -\sin x & \cos x\end{bmatrix} = \cos^2 x+\sin^2 x = 1. \end{cases}$

$$y_{p_1} = u_1 y_1 + u_2 y_2 \quad \text{where} \quad u_1 = -\int \frac{y_2 \cdot F}{W[y_1,y_2]}dx = -\int \frac{\sin x \cdot \sec x}{1}dx$$

$$\Rightarrow u_1 = -\int \tan x\, dx = -\ln|\sec x| = \ln|\cos x|$$

$$u_2 = \int \frac{y_1 \cdot F}{W[y_1,y_2]}dx = \int \frac{\cos x \cdot \sec x}{1}dx = \int dx = x.$$

$$\Rightarrow y_{p_1} = \ln|\cos x| \cdot \cos x + x \cdot \sin x.$$

$$\left.\begin{aligned} y_{p_2} &= Ae^{2x} \ \{1\} \\ y_{p_2}' &= 2Ae^{2x} \ \{0\} \\ y_{p_2}'' &= 4Ae^{2x} \ \{1\} \end{aligned}\right\} \Rightarrow Ae^{2x}(1+4) = 5Ae^{2x} = 10e^{2x} \Rightarrow A=2.$$

Sol:

$$y(x) = C_1\cos x + C_2\sin x + \ln|\cos x| \cdot \cos x + x\sin x + 2e^{2x}.$$

e) $16x^2y''+40xy'+9y=0;\ y(1)=2;\ y'(1)=-2$

Homogeneous:

$$\left.\begin{aligned} y &= x^r \quad \{9\} \\ y' &= rx^{r-1} \quad \{40x\} \\ y'' &= (r^2-r)x^{r-2} \quad \{16x^2\} \end{aligned}\right\} = x^r\left(16(r^2-r)+40r+9\right)=0$$

$$\Rightarrow \quad 16r^2-16r+40r+9=0$$

$$\underbrace{16r^2+24r+9}_{(4r+3)^2}=0. \Rightarrow$$

$$(4r+3)^2=0 \Rightarrow r=-\frac{3}{4},\ -\frac{3}{4}$$

$$y(x)=x^{-\frac{3}{4}}\left[c_1+c_2\ln x.\right].$$

$$y(1)=c_1=2$$

$$y'(x)=-\frac{3}{4}x^{-7/4}\left[c_1+c_2\ln x\right]+x^{-\frac{3}{4}}\left[\frac{c_2}{x}\right].$$

$$y'(1)=-\frac{3}{4}\left[c_1\right]+c_2=-2$$

$$c_2=\frac{3}{4}(2)-2=\frac{3}{2}-2=\frac{-1}{2}$$

Sol:

$$y(x)=x^{-\frac{3}{4}}\left[2-\frac{1}{2}\ln(x)\right],$$

f) : $x^2y'' - 4xy' + 6y = x^4\sin x$

Homogeneous: $y = x^r \quad \{6\}$

$y' = rx^{r-1} \quad \{-4x\}$

$y'' = (r^2-r)x^{r-2} \quad \{x^2\}$

$\Rightarrow x^r\left[r^2 - r - 4r + 6\right] = 0 \Rightarrow r^2 - 5r + 6 = 0 \rightarrow (r-2)(r-3) = 0$

$r = 2, 3$

$$y_h = c_1x^2 + c_2x^3 \qquad \left\{\begin{aligned} y_1 &= x^2 \\ y_2 &= x^3 \end{aligned}\right\} \Rightarrow W[y_1, y_2] = \det\begin{bmatrix} x^2 & x^3 \\ 2x & 3x^2 \end{bmatrix} = 3x^4 - 2x^4 = x^4$$

$y_p = u_1y_1 + u_2y_2$: $\quad F = \dfrac{x^4\sin x}{x^2} = x^2\sin x$.

where $u_1 = -\displaystyle\int \frac{y_2 \cdot F}{W}dx = -\int \frac{x^3 \cdot x^2 \sin x}{x^4}dx = \int x\sin x\,dx$

x	$\sin x$
1	$-\cos x$
0	$-\sin x$

$\Rightarrow u_1 = -x\cos x + \sin x$.

$$u_2 = \int \frac{y_1 \cdot F}{W}dx = \int \frac{x^2 \cdot x^2\sin x}{x^4}dx = \int \sin x\,dx = -\cos x$$

$\Rightarrow$ General Sol^n:

$$y = y_h + y_p = c_1x^2 + c_2x^3 + (\sin x - x\cos x)x^2 - \cos x\,(x^4).$$

2. A mass – spring system is governed by the IVP

$4y''+12y'+13y=0;\ y(0)=1;\ y'(0)=-1$ (15pts)

a) Find the solution to the IVP, (in phase – amplitude form)

$$P(\lambda) = 4\lambda^2+12\lambda+13 = \underbrace{(2\lambda)^2+2(2\lambda)\cdot 3+9}+4 = 0$$

$$\Rightarrow (2\lambda+3)^2 = -4 \Rightarrow \lambda = -\tfrac{3}{2} \pm i.$$

$$\Rightarrow y(t) = Ae^{-\frac{3}{2}t}\cos(t-\delta) \Rightarrow y(0) = A\cos(-\delta) = A\cos\delta = 1.$$

$$y'(t) = Ae^{-\frac{3}{2}t}\left[-\tfrac{3}{2}\cos(t-\delta) - \sin(t-\delta)\right] \Rightarrow y'(0) = A\left[-\tfrac{3}{2}\cos(-\delta) - \sin(-\delta)\right] = -1$$

$$\Rightarrow -\tfrac{3}{2}A\cos\delta + A\sin\delta = -1 \Rightarrow A\sin\delta = -1+\tfrac{3}{2} = \tfrac{1}{2}$$

$$A^2 = 1+\tfrac{1}{4} = \tfrac{5}{4} \Rightarrow A = \tfrac{\sqrt{5}}{2}$$

$$\Rightarrow \begin{cases} A\cos\delta = 1 \\ A\sin\delta = \frac{1}{2} \end{cases} \Rightarrow \delta \in QI \Rightarrow \tan\delta = \tfrac{1}{2} \Rightarrow \delta = \tan^{-1}\left(\tfrac{1}{2}\right) = 0.464$$

$$\Rightarrow y(t) = \frac{\sqrt{5}}{2}e^{-\frac{3}{2}t}\cos(t-0.464)$$

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b) Find the time at which the mass first crosses the equilibrium position.

1st - time, it crosses the equilibrium: $\Rightarrow t-0.464 = \begin{cases} \frac{\pi}{2} \\ -\frac{\pi}{2} \end{cases} \Rightarrow t = \begin{cases} \frac{\pi}{2}+0.464 = 2.03 \\ -\frac{\pi}{2}+0.464 = -1.107 \end{cases}$

$\Rightarrow$ $\underline{t = 2.03 \text{ seconds.}}$

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c) Estimate the time t for which $|y(t)| < \frac{1}{100}$

$$|y(t)| = \left|\frac{\sqrt{5}}{2}e^{-\frac{3}{2}t}\cos(t-0.464)\right| \le \frac{\sqrt{5}}{2}e^{-\frac{3}{2}t} \le \frac{1}{100}$$

$$\Rightarrow e^{-\frac{3}{2}t} \le \frac{1}{50\sqrt{5}} \Rightarrow -\frac{3}{2}t \le \ln\left(\frac{1}{50\sqrt{5}}\right)$$

$$\Rightarrow t \ge -\frac{2}{3}\ln\left(\frac{1}{50\sqrt{5}}\right) = 3.14 \text{ sec.}$$

3. A mass – spring system is governed by the IVP:

$3y''+11y'+10y=0;\ y(0)=1;\ y'(0)=-1$ (15 pts)

a) Find the solution to the IVP.

$p(\lambda)=3\lambda^2+11\lambda+10=(3\lambda+5)(\lambda+2)=0 \Rightarrow \lambda=-\frac{5}{2},-2.$

$$y(t)=c_1e^{-\frac{5}{2}t}+c_2e^{-2t}$$

$y(0)=c_1+c_2=1$

$y'(0)=-\frac{5}{2}c_1-2c_2=-1 \Rightarrow$

$4c_1+4c_2=4$

$-5c_1-4c_2=-2$

$-c_1=2 \Rightarrow c_1=-2$

$\Rightarrow c_2=1+2=3$

$$y(t)=-2e^{-\frac{5}{2}t}+3e^{-2t}.$$

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b) Find the time at which the mass first crosses the equilibrium position.

Cross the equilibrium $\Rightarrow$ $y(t)=-2e^{-\frac{5}{2}t}+3e^{-2t}=0. \Rightarrow \frac{3e^{-2t}}{3e^{-\frac{5}{2}t}}=\frac{2e^{-\frac{5}{2}t}}{3e^{-\frac{5}{2}t}}. \Rightarrow e^{\frac{1}{2}t}=\frac{2}{3}$

$\Rightarrow \frac{1}{2}t=\ln\left(\frac{2}{3}\right) \Rightarrow t=2\ln\left(\frac{2}{3}\right)\approx-0.81.$

$\Rightarrow$ The mass will never cross the equilibrium position.

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c) Estimate the time t for which $|y(t)|<\frac{1}{100}$

$$|y(t)|=\left|-2e^{-\frac{5}{2}t}+3e^{-2t}\right|<\frac{1}{100}.=0.01$$

Pick $t=2 \Rightarrow \left|-2e^{-5}+3e^{-4}\right|=0.04$

$\boxed{t=6 \Rightarrow \left|-2e^{-15}+3e^{-12}\right|=0.000017}$

$t\geq 6$ will work.

4. A 4 – pound weight is attached to a spring whose constant is 2 lb/ft. The medium offers a resistance to the motion of the weight numerically equally to the instantaneous velocity (i.e. c = 1). If the weight is released from a point 1 foot above the equilibrium position with a downward velocity of 8 ft/s,

a) Determine the time at which the weight passes through the equilibrium position.

b) Find the time at which the weight attains its extreme displacement from the equilibrium position. What is the position of the weight at this instant?

(15pts)

Sol: Weight = force: $F = m \cdot g \Rightarrow 4 = m \cdot 32 \Rightarrow m = \frac{1}{8}$

Spring constant $k = 2$; damping force: $c = 1$.

$$my'' + cy' + ky = 0 \Rightarrow \frac{1}{8}y'' + y' + 2y = 0 \Rightarrow y'' + 8y' + 16y = 0 \quad \begin{cases} y(0) = -1 \\ y'(0) = 8 \end{cases}$$

$$\Rightarrow p(\lambda) = \lambda^2 + 8\lambda + 16 = (\lambda+4)^2 = 0 \Rightarrow \lambda = -4, -4$$

$$y(t) = (c_1 + c_2 t)e^{-4t} \Rightarrow \begin{cases} y(0) = c_1 = -1 \\ y'(t) = e^{-4t}(-4c_1 - 4c_2 t + c_2) \\ y'(0) = -4c_1 + c_2 = 8 \Rightarrow c_2 = 4. \end{cases}$$

$$y(t) = (-1 + 4t)e^{-4t}.$$

a) 1st time, the mass cross the equilibrium $\Rightarrow y(t) = 0$

$$\Rightarrow (-1+4t)e^{-4t} = 0 \Rightarrow t = \frac{1}{4} = 0.25 \text{ sec}.$$

b) Extreme displacement $\Rightarrow$ maximum $\Rightarrow y'(t) = 0$.

$$y' = e^{-4t}(4 - 16t + 4) = e^{-4t}(8 - 16t) = 0 \Rightarrow t = \frac{1}{2} \text{ sec.}$$

Extreme displacement $\Rightarrow y\left(\frac{1}{2}\right) = \left(-1 + 4\left(\frac{1}{2}\right)\right)e^{-4\left(\frac{1}{2}\right)}$

$$= e^{-2} = 0.135 \text{ ft}.$$

time : $t = 0.5$ sec.

position : $p(0.5) = 0.135$ ft.

5. The motion of a mass – spring system is described by the DE: $y''+12y'+35y=0$, where y is in feet and t in seconds. The mass is released from 1ft below its equilibrium position with an downward velocity $v_0 \geq 0$. Determine the condition of the initial velocity so that the mass crosses the equilibrium position at least once. (15 pts)

Sol: $p(\lambda) = \lambda^2 + 12\lambda + 35 = (\lambda+5)(\lambda+7) = 0 \Rightarrow \lambda = -5, -7$

$$y(t) = c_1 e^{-5t} + c_2 e^{-7t} \qquad \begin{cases} y(0) = 1 \\ y'(0) = v_0 \geq 0 . \end{cases}$$

$$7\big(y(0) = c_1 + c_2 = 1\big),$$
$$y'(0) = -5c_1 - 7c_2 = v_0$$
$$2c_1 = 7 + v_0$$

$$\Rightarrow c_1 = \frac{7+v_0}{2} \quad \& \quad c_2 = 1 - \frac{7+v_0}{2} = \frac{2-7+v_0}{2} = \frac{v_0-5}{2}$$

$$y(t) = \frac{7+v_0}{2} e^{-5t} + \frac{v_0-5}{2} e^{-7t} .$$

Cross the equilibrium $\Rightarrow$ $y(t) = \frac{7+v_0}{2} e^{-t} + \frac{v_0-5}{2} e^{-7t} = 0.$

$$\frac{7+v_0}{2} e^{-t} = \frac{5-v_0}{2} e^{-7t} \Rightarrow \frac{(7+v_0)e^{-t}}{(7+v_0)e^{-7t}} = \frac{(5-v_0)e^{-7t}}{(7+v_0)e^{-7t}} .$$

$$\Rightarrow e^{6t} = \frac{5-v_0}{7+v_0} . \Rightarrow t = \frac{1}{6} \ln\left(\frac{5-v_0}{7+v_0}\right),$$

to cross the equilibrium $\Rightarrow t > 0 \Rightarrow \frac{1}{6}\ln\left(\frac{5-v_0}{7+v_0}\right) > 0$

$$\Rightarrow \frac{5-v_0}{7+v_0} \geq 1 \Rightarrow \frac{5-v_0}{7+v_0} - 1 \geq 0$$

$$\frac{5-v_0-7-v_0}{7+v_0} \geq 0 \Rightarrow \frac{-2v_0-2}{7+v_0} \geq 0 \quad \text{b/c } 7+v_0 > 0$$

$$\Rightarrow -2v_0 - 2 \geq 0 \Rightarrow -2v_0 \geq 2$$

$$\boxed{v_0 \leq -1}$$ Not possible

$\Rightarrow$ The mass will not cross the equilibrium pt for $V_0 \geq 0$.

6. Solve the following system of DE: (15 pts)

a) $\begin{cases} \dfrac{dx_1}{dt} = x_1 + 2x_2 \\ \dfrac{dx_2}{dt} = 2x_1 - 2x_2 \end{cases}$; $x_1(0) = 1$ and $x_2(0) = 0$

$$\begin{array}{r} 2 \\ (D-1) \end{array} \begin{cases} (D-1)x_1 - 2x_2 = 0 \\ -2x_1 + (D+2)x_2 = 0 \end{cases}$$

$$\left[-4 + (D-1)(D+2)\right]x_2 = 0$$

$$\left(D^2 + D - 6\right)x_2 = 0$$

$$p(\lambda) = \lambda^2 + \lambda - 6 = 0$$

$$(\lambda+3)(\lambda-2) = 0 \Rightarrow \lambda = -3, 2$$

$$x_2 = C_1 e^{-3t} + C_2 e^{2t}$$

$$-2x_1 + (D+2)x_2 = 0 \Rightarrow 2x_1 = (D+2)\left(C_1 e^{-3t} + C_2 e^{2t}\right)$$

$$2x_1 = -3C_1 e^{-3t} + 2C_2 e^{2t} + 2C_1 e^{-3t} + 2C_2 e^{2t}$$

$$\begin{cases} x_1 = \frac{1}{2} C_1 e^{-3t} + 2C_2 e^{2t} \Rightarrow x_1(0) = \frac{1}{2} C_1 + 2C_2 = 1 \\ x_2 = C_1 e^{-3t} + C_2 e^{2t} \Rightarrow x_2(0) = C_1 + C_2 = 0 \end{cases}$$

$$\Rightarrow C_1 = -C_2$$

$$-\frac{1}{2}C_2 + 2C_2 = 1 \Rightarrow \frac{3}{2} C_2 = 1 \Rightarrow C_2 = \frac{2}{3} \,;\; C_1 = -\frac{2}{3}$$

Sol:
$$\begin{cases} x_1 = \frac{1}{2}\left(-\frac{2}{3}\right)e^{-3t} + 2\left(\frac{2}{3}\right)e^{2t} \\ x_2 = -\frac{2}{3}e^{-3t} + \frac{2}{3}e^{2t} \end{cases}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{e^{-3t}}{3}\begin{bmatrix} -1 \\ 2 \end{bmatrix} + \frac{2}{3}e^{2t}\begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

b) $\begin{cases} \dfrac{dx_1}{dt} = x_1 + x_2 + e^{2t} \\ \dfrac{dx_2}{dt} = 3x_1 - x_2 + 5e^{2t} \end{cases}$ $(D+1)$ $\Rightarrow \begin{cases} (D-1)x_1 - x_2 = e^{2t} \\ -3x_1 + (D+1)x_2 = 5e^{2t}. \end{cases}$

$$[(D+1)(D-1)-3]x_1 = (D+1)e^{2t} + 5e^{2t}.$$

$$(D^2-4)x_1 = 2e^{2t} + e^{2t} + 5e^{2t} = 8e^{2t}.$$

$$x_{1h}: \ p(\lambda) = \lambda^2 - 4 = 0 \Rightarrow \lambda = \pm 2.$$

$$\Rightarrow x_{1h} = C_1 e^{2t} + C_2 e^{-2t}.$$

$$\begin{rcases} \{-4\} \ x_{1p} = Ate^{2t} \\ \{0\} \ x'_{1p} = Ae^{2t}[2t+1] \\ \{1\} \ x''_{1p} = Ae^{2t}[4t+2+2] \end{rcases} \Rightarrow Ae^{2t}[-4t+4t+4] = 8e^{2t}$$

$$\Rightarrow 4A = 8 \Rightarrow A = 2$$

$$x_1 = x_{1h} + x_{1p} = C_1e^{2t} + C_2e^{-2t} + 2te^{2t}.$$

from: $(D-1)x_1 - x_2 = e^{2t}.$

$$\Rightarrow x_2 = (D-1)\left[C_1e^{2t} + C_2e^{-2t} + 2te^{2t}\right] - e^{2t}$$

$$= 2C_1e^{2t} - 2C_2e^{-2t} + 2e^{2t}(2t+1) - C_1e^{2t} - C_2e^{-2t} - 2te^{2t} - e^{2t}$$

$$= C_1e^{2t} - 3C_2e^{-2t} + 4te^{2t} + 2e^{2t} - 2te^{2t} - e^{2t}$$

$$= C_1e^{2t} - 3C_2e^{-2t} + 2te^{2t} + e^{2t}$$

Sol.:

$$\begin{cases} x_1 = c_1 e^{2t} + c_2 e^{-2t} + 2t e^{2t} \\ x_2 = c_1 e^{2t} - 3c_2 e^{-2t} + 2t e^{2t} + e^{2t} \end{cases}$$